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INTERACTION OF HYPERSONIC MULTIPHASE FLOWS

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The investigations of multiphase flows, pursued vigorously in recent years, stem from the practical importance of problems such as supersonic combustion, erosion of materials exposed to flow, various problems of chemical technology, etc.. These flows are also of great interest from the viewpoint of building high-enthalpy gasdynamic facilities [1], which would, in principle, offer modeling of the most important flight parameters of hypersonic vehicles. The basic gasdynamic problem in these areas is to arrange the process of mixing a group of solid (or liquid) particles, accelerated by a light gas, with the supersonic quasiauxiliary flow in which one can excite internal, particularly vibrational, degrees of freedom. The solution of the complete problem can be divided into a number of stages. The first problem is to accelerate solid particles to hypersonic speeds. When the mass ratios of the accelerating and accelerated components are close, the light-gas temperature must be low enough so that the vapors formed in the acceleration (in the case where the particles may vaporize) should not harm the carrier properties of the light gas. It is important to achieve maximum velocities of the solid particles and uniform distribution across the accelerating nozzle. The second task is to examine the mixing process with a view to minimizing perturbation associated with percolation of the particles, their dynamic motion and possible vaporization. Nonuniformities can arise in the flow from several causes: shock waves of various strengths, turbulent fluctuations, etc. To minimize perturbations one must, firstly, so choose the parameters of the interacting gas components and their encounter angle, so that a shock wave does not arise in one of the flows, which may be, e.g., air (Fig. 1). Such a shock wave, however, may be formed for another reason: Because of penetration and vaporization of particles additional perturbations arise, associated with the supply of mass, momentum and energy. Here the macroscopic parameters vary in the mixture. When one cannot achieve conditions for quasiauxiliary flow (i.e., the flow velocity of the gas into which the particles are introduced equal to the tangential component of the particle velocity), additional acceleration of the particles occurs in a certain layer, accompanied by dissipative irreversible processes. There may also be rapid relaxation of vibrational energy in the layer; in addition, the layer may be a source of additional wavelike perturbations.

Thus, both the acceleration of particles, and analysis of the processes occurring inside the zone where the particles mix with the gas stream, are important and independent tasks. We shall examine them in succession. It is known that one can obtain aerosols by using the phenomenon of condensation in a supersonic

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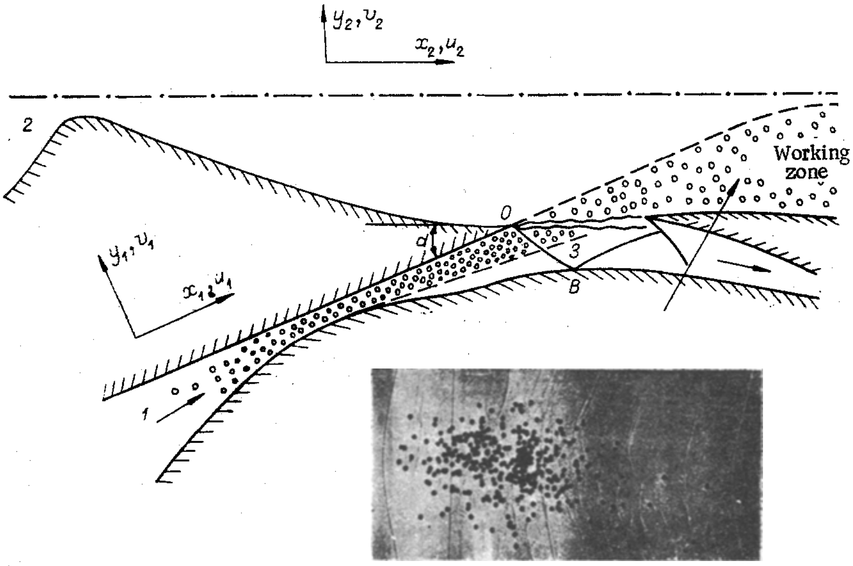

Fig. 1

gas expanding from a nozzle, but here the mass ratio between the solid (or liquid) and the gaseous phases is small, and the size of the particles formed is very small ($\hat{a}$ = 1 μm), in order to achieve deep enough penetration into the other stream, particularly into a stream with a vibrationally excited medium. Therefore, for particle acceleration it makes sense to use a light gas (hydrogen or helium) and obtain particles by injecting through an atomizer into the stagnation chamber of the accelerating nozzle, where the particles settle, are then entrained by the light gas, and accelerated through a supersonic nozzle of special shape. This principle of acceleration was suggested in [1] for creating promising high-enthalpy gasdynamic facilities. With reference to the conditions examined here the requirements are: Particles of a given size must make up the main part of the total number of particles formed in the stagnation chamber, and must be accelerated to velocities equal to the flow velocity of the vibrationally excited gas. The particles should not evaporate in the acceleration process to form vapors which would degrade the carrier properties of the light gas. These requirements are contradictory, in a certain sense, since for efficient acceleration one must increase the pressure and temperature of the light gas in the stagnation chamber, and simultaneously, an increase in temperature will lead to evaporation of the accelerated particles. The problem of accelerating solid particles with a light gas is substantially a two-dimensional one and is examined below.

The system of equations for mixing of a perfect gas with a constant specific heat ratio $\varkappa$ and fixed spherical particles distributed in size is described in the following form [2] (ν = 0; 1 for the planar and axisymmetric flows):

$$\frac{\partial\rho u}{\partial x}+\frac{\partial\rho v}{\partial y}=-\frac{\nu\rho v}{y}; \quad (1)$$

$$\frac{\partial(\rho u^2+p)}{\partial x}+\frac{\partial\rho uv}{\partial y}=-\frac{\nu\rho uv}{y}-\sum_{i=1}^{N}\hat{\rho}_i\hat{f}_{xi}; \quad (2)$$

$$\frac{\partial\rho uv}{\partial x}+\frac{\partial(\rho v^2+p)}{\partial y}=-\frac{\nu\rho v^2}{y}-\sum_{i=1}^{N}\hat{\rho}_i\hat{f}_{yi}; \quad (3)$$

$$\frac{\partial\rho u\omega}{\partial x}+\frac{\partial\rho v\omega}{\partial y}=-\frac{\nu\rho v\omega}{y}-\sum_{i=1}^{N}\rho_i\left(\hat{f}_{x_i}\hat{u}_i+\hat{f}_{y_i}\hat{v}_i\right)-\frac{c_0}{R}\sum_{i=1}^{N}\hat{\rho}_i q_i; \quad (4)$$

$$\frac{\partial\hat{\rho}_i u_i}{\partial x}+\frac{\partial\hat{\rho}_i\hat{v}_i}{\partial y}=-\frac{\nu\rho_i v_i}{y}; \quad (5)$$

$$\hat{u}_i\frac{d\hat{u}_i}{dx}=\hat{f}_{xi}=k_i(u-\hat{u}_i); \quad (6)$$

$$\hat{u}_i\frac{d\hat{v}_i}{dx}=\hat{f}_{yi}=k_i(v-\hat{v}_i); \quad (7)$$

$$\hat{u}_i\frac{d\hat{T}_i}{dx}=\hat{q}_i, \quad (8)$$

$$k_i = \beta \frac{c_{D_i}}{\hat{q}_i} \rho [(u - \hat{u}_i)^2 + (v - \hat{v}_i)^2]^{1/2}, \ \beta = \frac{3}{8} \frac{\rho_*}{\rho_0} \frac{r_*}{\hat{a}_*}, \ \omega = \frac{\varkappa}{\varkappa - 1} \frac{p}{\rho} + \frac{u^2 + v^2}{2}.$$

This system of equations may be reduced to dimensionless form as follows: All the linear dimensions, apart from particle radius, are referenced to the radius of the nozzle throat section (or the semiwidth of a planar nozzle), $r_* \equiv y_*$; the densities are referenced to ρ_*, the velocities are referenced to a_*, the pressure to $\rho_* a_*^2$; the temperatures are referenced to a_*^2/R, the components of acceleration of the i-th particle f_{x_i} and f_{y_i} are referenced to a_*^2/r_*, and the heat flux to a particle $\hat{q}_i$ is referenced to a_*^3/r_*. The parameters of all the particles are given the sign ∧, and the properties of the material are designated with a subscript 0. The particle radii are referenced to $a_* = 1\ \mu m$. The particle momentum and energy equations, Eqs. (6)-(8), are written in the form of characteristic ratios along a streamline.

The right sides of this system of equations, which describe the force and thermal interaction of the particles with the gas, and are suitable for arbitrary values of Knudsen number $\hat{Kn}_i = l/2\hat{a}_i$ (small particles may find themselves in free-molecular flow conditions even at the nozzle throat), were given in [2]. The expressions for $\hat{Kn}_i \ll 1$ (in the continuum flow regime) take into account the known semiempirical information on the coefficients for drag and heat transfer of a sphere as a function of Mach and Reynolds numbers, based on the relative velocity and particle diameter, and for $\hat{Kn}_i \gg 1$ they tend asymptotically to the respective rarefied gasdynamic formulas, containing momentum and energy accommodation coefficients for the molecules at the particle surface.

A numerical solution of this system of equations in the transonic part of the accelerating nozzle was obtained by the Godunov method using the program of [3], improved by allowing for stored interaction coefficients (in this program the particle dynamics equations are written in partial derivatives).

In a numerical investigation of the flow of a multiphase mixture, in the supersonic part of the nozzle the continuity equation, Eq. (5), is replaced by the algebraic ratio $\psi = \mathrm{const}$ along the particle trajectory; then the following definition of a stream function for the particles is introduced:

$$d\psi = 2y^\nu \hat{\rho}(\hat{u}dy - \hat{v}dx).$$

The system of equations describing the gas flow was solved by the method suggested by Kraiko and Ivanov; the equations of particle dynamics and heat balance in characteristic form, Eqs. (6)-(8), are valid along the trajectories, and were integrated with second-order accuracy. A more detailed description of the performance of this algorithm has been given in [4].

Below we give an example of a solution, obtained with the following values of the dimensionless parameters, typical of the pair – accelerating gas helium and solid particles CO_2: $\varkappa = 5/3$, $\omega = 0.647$,

$$R/c^0 = 1.54, \ \beta = \frac{3}{8} \frac{\rho_*}{\rho_0} \frac{y_*}{\hat{a}_*} = 0.35.$$

Since the mass spectrum of the CO_2 particles depends appreciably on the structural design of the dispersing device and the method of introducing the particles into the helium flow, the solution was carried out for a monodisperse suspension with $\hat{a}_i \equiv \hat{a} = 5\ \mu m$. The nozzle shape and some of the computed results are given in Fig. 2. The transonic solution was obtained in the region $0 \le x \le 7$; the values found for the mixing parameters in the plane $x = 5$, which lies in the supersonic flow region, are used as initial conditions for subsequent integration of the system (1)-(8). In the region $5 \le x \le 7$ we checked for agreement of the results obtained by the two programs.

Figure 2 shows the distribution of mixing parameters along the nozzle plane of symmetry. It can be seen that, for a signiticant gas stagnation pressure $p_0 = 7$ MPa and $T_0 = 250°K$, $y_* = 10^{-3}$ m, particles of the carbon dioxide gas are accelerated to a considerable velocity $\hat{u}$. With a small semiopening angle for the supersonic part of the nozzle ($\alpha_+ = 5°$) the layer of pure gas lying between the wall Γ and the separatrix S is quite thin, and the distribution of parameters of the particle gas across the nozzle is almost uniform (the gas operates quite well).

The dot–dash lines in Fig. 2 correspond to the flow parameters of "pure" helium without particles. It can be seen that the back action of the particles on the gas leads to deceleration and heating, and to shift of the line $M = 1$ towards the accelerating part of the nozzle.

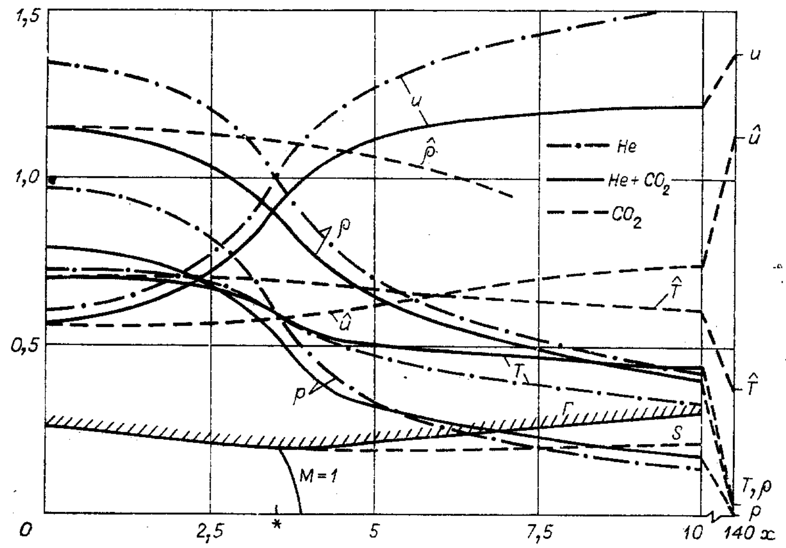

Fig. 2

Analysis of other calculations, corresponding to different values of p_0, T_0, α, show that, from the viewpoint of efficient acceleration of particles by the light gas and of retention of the "carrier properties" of the latter, it makes sense to increase p_0, reduce T_0 and α_+, and elongate the transonic part of the nozzle in which the most intense interaction of phases occurs. The question of possible limits for variation of the gas temperature T_0 which are favorable from the viewpoint of maximum particle acceleration can be resolved, in the problem considered, after one includes processes whereby the particles are vaporized into the light-gas carrier.

The second task is the interaction of two supersonic gas streams, and is solved in the two-dimensional inviscid flow formulation. We consider that the particles pass freely through the shock wave in the light gas and through the tangential discontinuity, and we neglect the influence of particles on the process of interaction of the two streams.*

We shall carry out calculations, assuming uniform parameters for the incident stream of light gas (we consider helium, whose parameters are denoted by the subscript 1) and hydrogen (subscript 2). Then the solution investigated is self-similar in the variable $\theta = y_1/x_1$ (see Fig. 1).

In order that flow 2 should not be perturbed during interaction and that the tangential discontinuity from the point 0 should be directed along the velocity vector of the nitrogen, the pressure p_3 in helium behind the shock wave should be equal to the pressure in the nitrogen p_2.

With the given parameters of the nitrogen flow and the number M_1 in helium one must choose a value ρ_1/ρ_2 so that the condition $p_3 = p_2$ is obtained for various encounter angles α between the two flows. We denote by W_1 the velocity ratio of the particles and the helium on the axis of a two-dimensional nozzle

$$W_1 = \hat{u}/u_1, \tag{9}$$

and by W_2 the ratio of the longitudinal velocity component of the particles to the nitrogen velocity in the vicinity of the point 0

$$W_2 = \hat{u}\cos\alpha/u_2. \tag{10}$$

From the equality of the particle velocity $\hat{u}$ in Eqs. (9) and (10) we obtain a relation between the velocities of the gases

$$u_2 W_2 = u_1 W_1 \cos\alpha.$$

*The inverse effect of particles on the gas parameters in the mixing zone can be taken into account later using the method proposed above, in which the acceleration of the particles by the light gas is determined.

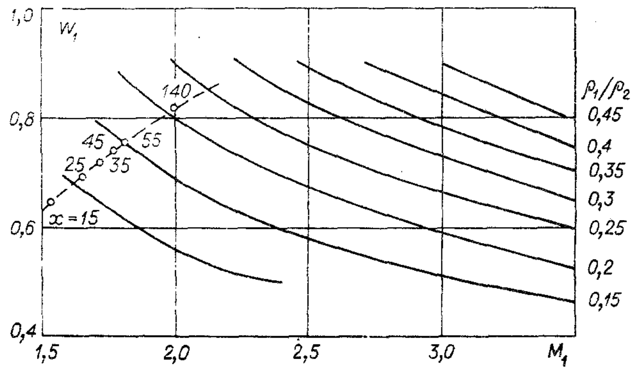

Fig. 3

We must stress once more that, according to our hypothesis, the particles do not affect the flow configuration. The introduction of the quantities W_1, W_2 and their ratio $N = W_1/W_2 = u_2/u_1 \cos \alpha$ makes it simple and convenient to parameterize the problem. For a given nitrogen velocity u_2 an increase in the parameter N leads to a decrease in the quantity u_1.

For each specific variation of the two "colliding" flows the problem reduces to solving a two-parameter system of transcendental algebraic equations by the method of iteration [5].

Figure 3 shows the relation $\varkappa_1 = 1.667$, $\varkappa_2 = 1.4$, $M_2 = 7$, $W_2 = 0.4$, $\alpha = 10°$ for constant values of the density ratio $W_1 = f(M_1)$ for the given parameters ρ_1/ρ_2. The broken line shows the relation $W_1 = f(M_1)$ for the actual helium nozzle calculated in the first part of the present paper.

We now consider the third problem. We shall analyze the processes in the mixing zone, assuming that the pressure can vary in the range $(1\text{-}0.001) \cdot 10^5$ Pa, the spherical particle diameter in the range $(1\text{-}10^2 \cdot 10^{-6}$, and the mass ratio of solid and gaseous phases per unit volume in the range from 1 to 100%. In the interaction of the particles and the gas one can find both continuum flow conditions and transitional conditions, including conditions close to free molecular. We shall consider the continuum regime. In the absence of quasiauxiliary conditions the gas velocity is supersonic relative to the particles. Around each of the particles there arises a shock wave, and these interact with one another. In principle these waves may overlap and can even create a common shock front ahead of the entire group of particles. The question arises as to whether the group wave is actually generated under such conditions, and conditions under which it is not generated. Figure 1 shows a photograph of the flow over a group of flying pellets, obtained by Krasil'shchikov and Gulyaev in a ballistic range. This photograph is a kind of model of what may actually occur in the situation that we are investigating. The flow picture is quite complex. Figure 4 shows a model of the interaction in the case of two particles, located in a plane normal to the relative velocity vector, at different distances apart. It is clear that a common bow wave is formed in the case when there is overlapping of the transonic zones of the shock layers. We take this information as a basis for further analysis. Let the particles have a diameter d, let the mean distance between them be l, and let l^* be the characteristic size of the transonic region. Then, depending on the ratio between the scales $\lambda^* \sim l^*/d$ and $\lambda = l/d$ we can judge as to the presence or absence of a group wave. The ratio l^*/d is increased with decrease of the relative Mach number M of the particle motion, i.e., with its acceleration (Fig. 4), but at the same time there is a decrease in the shock-front intensity. The function $(l/d)^* = f(M)$ is determined from numerical calculations of [6]. Figure 4 shows this dependence in the case where the ratio of the specific heats is $\varkappa = 1.4$. Allowing for the three-dimensional distribution of particles the corresponding scale λ^* will differ from the case of the two-dimensional and symmetric configuration of Fig. 4, and it will be considerably more difficult to determine a criterion for formation of a common wave in this case. We therefore consider the actual ratios between λ^* and λ in the range of pressure and particle concentration of interest to us. If we consider that the particles are identical in size, we can derive the result that $\lambda \sim (2M_T/\rho_T)^{-1/3}$, where M, ρ_T are the common mass and density of the solid phase per unit volume. Thus, the quantity λ does not depend on the particle size. For $\rho_T \sim 1$ g/cm^3 in Table 1 we show values of λ for various values of pressure and mass ratio of the solid gas phases.

It follows from the data of Fig. 4 that $\lambda^*_{max} \approx 5$; therefore, the parameter $\lambda \gg \lambda^*$ because of the large density of the solid phase, in practically all cases, and where there is no reverse influence of particles on the gas, one would not expect a group shock wave to occur. However, in actual fact, wave perturbations can

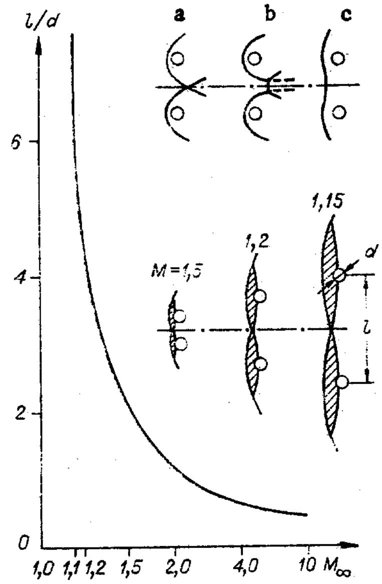

Fig. 4

TABLE 1

M_T/M_{N_2} \ p, atm	1	0,1	0,01	0,001
1	7	15	30	70
0,1	15	30	70	150
0,01	30	70	150	310

occur due to the evaporating particles altering the macroscopic parameters of the medium into which they penetrate. For this reason it is very important to analyze the phenomena in the zone where the particles mix with a vibrationally excited gas.

In experiments, as a rule, the total mass flow is controlled (i.e., the ratio between the solid and gaseous components); it is practically impossible to assign the required particle-size distribution beforehand. Therefore, the heavy particles begin to play a key role in the total mass of solid phase supplied to the flow, even if their relative concentration is low.

Figure 5 shows one possible particle-size distribution (normalized to unity), typical for clouds and rocket engine nozzles with metallized fuel [7] (Curve 1)

$$f(\hat{a}) = \frac{d\hat{n}}{\hat{n}d\hat{a}}\frac{\beta^{\alpha}\hat{a}^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta\hat{a}}.$$

The moments of this distribution (mean, and mean-cube radii and dispersion) that interest us have the form

$$\langle a\rangle = \frac{\alpha}{\beta},\ \langle a^3\rangle^{1/3} = \left[\frac{\alpha(\alpha+1)(\alpha+2)}{\beta^3}\right]^{1/3},\ D = \frac{\alpha}{\beta^2}.$$

The distribution presented corresponds to $\langle a\rangle = 5\ \mu m$, $D = 12.5\ \mu m^2$ ($\alpha = 2$, $\beta = 0.4$). Figure 5 also shows the relative particle density distribution by size $(1/\rho)d\hat{\rho}/d\hat{a} = \hat{a}^3 f(\hat{a})/\langle\hat{a}^3\rangle$ (curve 2). It can be seen, in particular, that although the number of heavy particles (e.g., $\langle\hat{a}\rangle > 4\ \mu m$, $\langle\hat{a}\rangle \le 20\ \mu m$) is small, their contribution to the mass characteristic of the particle cloud is appreciable (to illustrate this we can compare the shaded area under these curves, bounded by the particle sizes $\langle a\rangle \pm \sqrt{D}$). This fact becomes especially important for distributions of the normal-logarithmic type, which have a "heavy tail," and, as was shown in [8], are typical of pellet material. Therefore, it is very important to achieve the most economical mass spectrum of injected particles of the required size. Otherwise it may turn out that the mixture (e.g., $CO_2 + N_2$) will not contain

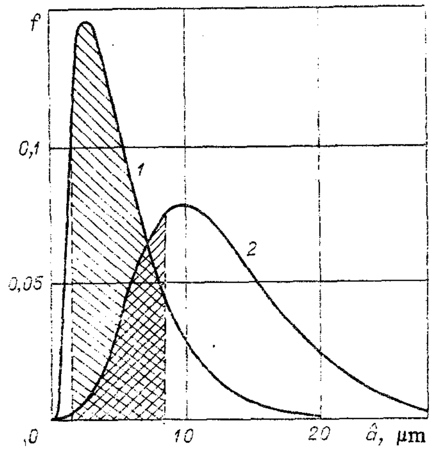

Fig. 5

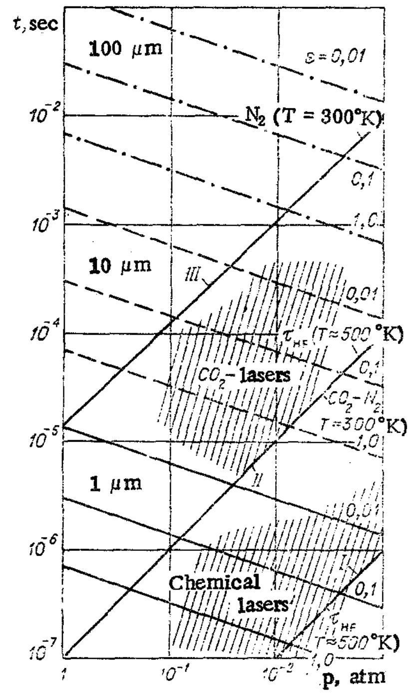

Fig. 6

enough carbon dioxide vapor, since the heavy particles pass through the stream of vibrationally excited gas with practically no evaporation, and to accelerate them requires an additional amount of light gas.

An important question is the mixing of the products of the solid phase among the molecules of the vibrationally excited medium. The basic mechanism leading to mixing is laminar or turbulent diffusion.

Because of the small particle size the generation of turbulence in the conditions considered is possible only in the auxiliary wakes at large distances from the particles. If conditions for quasiauxiliary flow do not hold, the acceleration of particles must lead to heating of the flow. Then, if the auxiliary wakes behind the particles do not intersect, in a time characteristic of the problem (e.g., the vibrational relaxation time), the flow heating will be local and cold gas will flow between the particles. It is evident that then one cannot obtain a uniform medium in a large enclosure. Figure 6 shows curves of the characteristic time for t_D for laminar diffusion, at the end of which there is mixing of the components, depending on the pressure, the particle size, and the particle-mass percentage concentration ε relative to the flowing gas; the numbers I–III show the characteristic times for exciting the upper vibrational levels of the HF molecules [9], IV–V show the vibrational relaxation in an $N_2 + CO_2$ [00°1 + v(1)] mixture [10] and deactivation of HF, and also the characteristic time for vibrational deactivation of the N_2 molecules [10]. The data in Fig. 6 are evidence that the mixing time, i.e., the time to prepare the medium for relatively loarse particles of d ~ 10-100 μm, which can penetrate to sufficient depth into the flow, is large. This situation may be improved by turbulent diffusion, which leads, however, to less uniform mixing.

In two-phase mixing complex heat and mass transfer processes occur between the particles and the gas, accompanied by evaporation under the conditions of the vibrationally excited medium, flow of particles over a wide range of Knudsen numbers ranging from continuum to free molecular, processes of homogeneous and heterogeneous vibrational relaxation, etc.. Here one requires to achieve maximum uniformity in the distribution of products of the vapor phase amongst molecules of the basic flow, with minimum perturbation of the flow. Above we considered only some aspects, associated with macroprocesses occurring in the interaction of multiphase supersonic flow. However, to obtain a uniform medium in large volumes with optimal characteristics we must solve a number of "internal" problems, associated with deep pentration of various types of particles, their lifetime on the scale of the basic flow time period associated with vibrational relaxation processes, determination of the concentration profile for the vapor of the additive in the flow, etc. The investigation of these questions is also important for the solution of other gasdynamic problems, particularly those mentioned in [11].

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